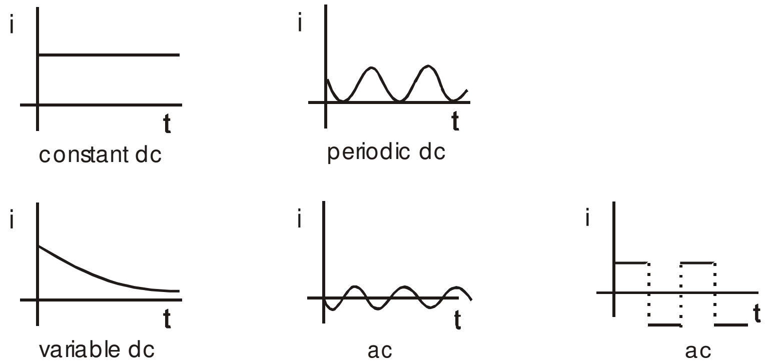
ALTERNATING CURRENT

1. AC AND DC CURRENT :

A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).

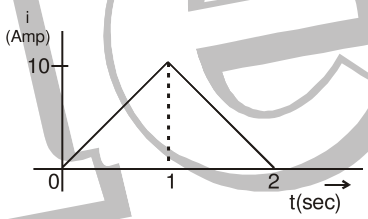


If a function suppose current, varies with time as $i = I_m \sin(\omega t+\phi)$, it is called sinusoidally varying function. Here I_m is the peak current or maximum current and i is the instantaneous current. The factor $(\omega t+\phi)$ is called phase. ω is called the angular frequency, its unit rad/s. Also $\omega = 2\pi f$ where f is called the frequency, its unit s^{-1} or Hz. Also frequency f = 1/T where T is called the time period.

2. AVERAGE VALUE :

Average value of a function, from t_1 to t_2, is defined as $\langle f\rangle = \dfrac{\int_{t_1}^{t_2} f\,dt}{t_2 - t_1}$. We can find the value of $\int_{t_1}^{t_2} f\,dt$ graphically if the graph is simple. It is the area of f-t graph from t_1 to t_2.

Ex. 1 Find the average value of current shown graphically, from t = 0 to t = 2 sec.



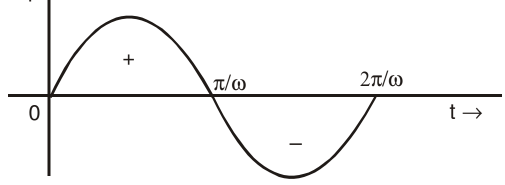
Sol. From the i – t graph, area from t = 0 to t = 2 sec

$= \frac{1}{2} \times 2 \times 10 = 10$ Amp. sec.

$\therefore$ Average Current $= \frac{10}{2} = 5$ Amp.

Ex. 2 Find the average value of current from t = 0 to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

Sol. $\langle i\rangle = \dfrac{\int_0^{2\pi/\omega} I_m \sin\omega t\,dt}{\frac{2\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1-\cos\omega\frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0$



It can be seen graphically that the area of i – t graph of one cycle is zero.

$\therefore$ < i > in one cycle = 0.

Ex. 3 Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero

Sol.

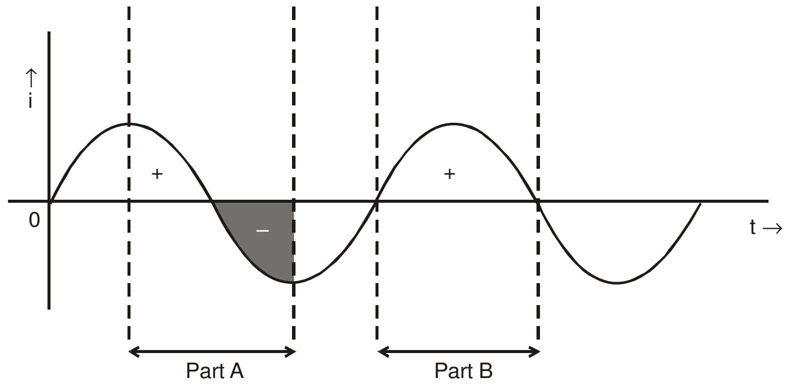


Figure shows two parts A and B, each half cycle. In part A we can see that the net area is zero

$\therefore$ $< i >$ in part A is zero.

In part B, area is positive hence in this part $< i > \neq 0$.

Ex. 4 Find the average value of current $i = I_m \sin \omega t$ from (i) $t = 0$ to $t = \frac{\pi}{\omega}$ (ii) $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$.

Sol. (i) $<i> = \dfrac{\int_0^{\pi/\omega} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{\pi}{\omega}\right)}{\frac{\pi}{\omega}} = \dfrac{2I_m}{\pi}$ (ii) $<i> = \dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = 0.$

Ex. 5 Current in an A.C. circuit is given by $i = 2\sqrt{2}\ \sin(\pi t + \pi/4)$, then the average value of current during time $t = 0$ to $t = 1$ sec is:

Sol. $<i> = \dfrac{\int_0^1 i\, dt}{1} = \mathbf{2}\sqrt{2} \int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right) = \dfrac{4}{\pi}$ **Ans.**

3. ROOT MEAN SQUARE VALUE:

Root Mean Square Value of a function, from t_1 to t_2, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

Ex. 6 Find the rms value of current from $t = 0$ to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

Sol. $i_{rms} = \sqrt{\dfrac{\int_0^{2\pi/\omega} {I_m}^2 \sin^2 \omega t dt}{\frac{2\pi}{\omega}}} = \sqrt{\dfrac{{I_m}^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

Ex. 7 Find the rms value of current $i = I_m \sin \omega t$ from (i) $t = 0$ to $t = \frac{\pi}{\omega}$ (ii) $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$.

Sol. (i) $i_{rms} = \sqrt{\dfrac{\int_0^{\pi/\omega} I_m{}^2 \sin^2 \omega t dt}{\dfrac{\pi}{\omega}}} = \sqrt{\dfrac{I_m{}^2}{2}} = \dfrac{I_m}{\sqrt{2}}$ (ii) $<i> = \sqrt{\dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m^2 \sin^2 \omega t\, dt}{\dfrac{\pi}{\omega}}} = \sqrt{\dfrac{I_m{}^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

Note: The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same. From the above two examples note that for sinusoidal functions **rms value** (Also called **effective value**)

$$= \frac{\text{peak value}}{\sqrt{2}} \quad \text{or} \quad \mathbf{I_{rms}} = \frac{\mathbf{I_m}}{\sqrt{\mathbf{2}}}$$

Ex. 8 Find the effective value of current i = 2 sin 100 πt + 2 cos (100 πt + 30º).

Sol. The equation can be written as i = 2 sin 100 πt + 2 sin (100 πt + 120º)
so phase difference ϕ = 120º

$$I_m)_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

$$= \sqrt{4 + 4 + 2 \times 2 \times 2\left(-\frac{1}{2}\right)}$$ = 2, so effective value or rms value = $2/\sqrt{2} = \sqrt{2}$ A

Ques. The peak voltage in a 220 V AC source is
(A) 220 V (B) about 160 V (C) about 310 V (D) 440 V

Ans. (C)

Ques. An AC source is rated 220 V, 50 Hz. The average voltage is calculated in a time interval of 0.01 s. It
(A) must be zero (B*) may be zero (C) is never zero (D) is (220/√2)V

Ans. (B)

Ques. Find the effective value of current i = 2 + 4 cos 100 πt.

Ans. **$2\sqrt{3}$ A**

Ques. The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value. How long will the current take to reach the peak value starting from zero?

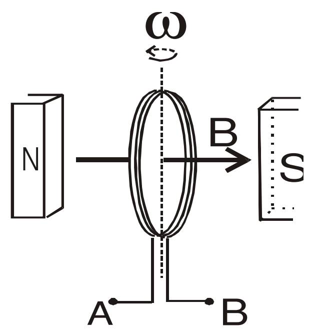
Ans. $i = \dfrac{T}{4} = \dfrac{1}{240}$ s

Ques. An alternating current havingpeak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is
(A) 14 A (B) about 20 A (C) 7 A (D) about 10 A

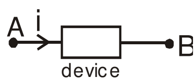
Ans. (D)

4. AC SINUSOIDAL SOURCE:

Figure shows a coil rotating in a magnetic field. The flux in the coil changes as φ = NBA cos (ωt + ϕ). Emf induced in the coil, from Faraday's law is $\dfrac{-d\varphi}{dt}$ = N B A ω sin (ωt + ϕ). Thus the emf between the points A and B will vary as E = E_0 sin (ωt +ϕ) .The potential difference between the points A and B will also vary as V = V_0 sin (ωt + ϕ). The symbolic notation of the above arrangement is A–(~)–B. We do not put any + or – sign on the AC source.



5. POWER CONSUMED OR SUPPLIED IN AN AC CIRCUIT:

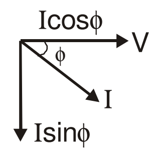


Consider an electrical device which may be a source, a capacitor, a resistor, an inductor or any combination of these. Let the potential difference be $v = V_A - V_B = V_m \sin\omega t$. Let the current through it be $i = I\sin(\omega t + \phi)$. Instantaneous power P consumed by the device $= v\,i = (V_m \sin\omega t)\,(I_m \sin(\omega t + \phi))$

$$\text{Average power consumed in a cycle} = \frac{\int_0^{\frac{2\pi}{\omega}} P\,dt}{\frac{2\pi}{\omega}} = \frac{1}{2} V_m I_m \cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi = V_{rms} I_{rms} \cos\phi.$$

Here $\cos\phi$ is called **power factor**.



Note : $I\sin\phi$ is called "wattless current".

Ex. 9 When a voltage $v_s = 200\sqrt{2}\,\sin(\omega t + 15^\circ)$ is applied to an AC circuit the current in the circuit is found to be $i = 2\sin(\omega t + \pi/4)$ then average power concsumed in the circuit is

(A) 200 watt (B) $400\sqrt{2}$ watt (C) $100\sqrt{6}$ watt (D) $200\sqrt{2}$ watt

Sol. $P_{av} = v_{rms}\ I_{rms} \cos\phi$

$= \dfrac{200\sqrt{2}}{\sqrt{2}}\ \dfrac{2}{\sqrt{2}} \cdot \cos(30^\circ) = 100\sqrt{6}$ watt

Ques. Find the average power concumed in the circuit if a voltage $v_s = 200\sqrt{2}\,\sin\omega t$ is applied to an AC circuit and the current in the circuit is found to be $i = 2\sin(\omega t + \pi/4)$.

Ans. 200W

6. SOME DEFINITIONS:

The factor **cos φ** is called **Power factor**.

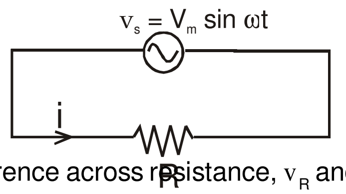
I_m **sin φ** is called **wattless current**.

Impedance Z is defined as $Z = \dfrac{V_m}{I_m} = \dfrac{V_{rms}}{I_{rms}}$

ωL is called **inductive reactance** and is denoted by X_L.

$\dfrac{1}{\omega C}$ is called **capacitive reactance** and is denoted by X_C.

7. PURELY RESISTIVE CIRCUIT:



Writing KVL along the circuit,

$v_s - iR = 0$

or $i = \dfrac{v_s}{R} = \dfrac{V_m \sin\omega t}{R} = I_m \sin\omega t$

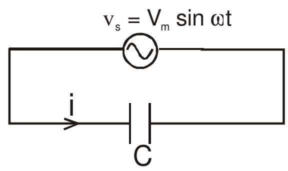
⇒ We see that the phase difference between potential difference across resistance, v_R and i_R is 0.

$I_m = \dfrac{V_m}{R}$

$I_{rms} = \dfrac{V_{rms}}{R}$

$\langle P \rangle = V_{rms} I_{rms} \cos\phi = \dfrac{V_{rms}^{\ 2}}{R}$

8. PURELY CAPACITIVE CIRCUIT:

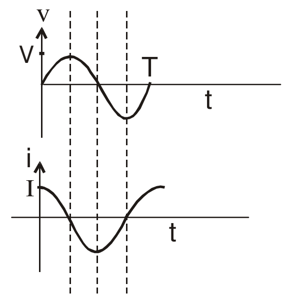


Writing KVL along the circuit,

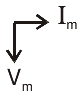
$$v_s - \frac{q}{C} = 0$$

or $i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = \frac{d(CV_m \sin\omega t)}{dt} = CV_m\omega\cos\omega t = \frac{V_m}{1/\omega C}\cos\omega t = \frac{V_m}{X_C}\cos\omega t = I_m\cos\omega t.$

$X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is ohm Ω.



From the graph of current versus time and voltage versus time , it is clear that current attains its peak value at a time $\frac{T}{4}$ before the time at which voltage attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference

$= \omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. i_C leads v_C by $\pi/2$ Diagrammatically (phasor diagram) it is represented as .

Since $\phi = 90º$, $\langle P \rangle = V_{rms} I_{rms} \cos\phi = 0$

Ques. A capacitor acts as an infinite resistance for
(A) DC (B) AC (C) DC as well as AC (D) neither AC nor DC

Ans. (A)

Ex. 10 An alternating voltage $E = 200\sqrt{2}\sin(100\,t)$ V is connected to a 1μF capacitor through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

Sol. Comparing $E = 200\sqrt{2}\sin(100\,t)$ with $E = E_0 \sin\omega t$ we find that,

$E_0 = 200\sqrt{2}$ V and $\omega = 100$ (rad/s)

So, $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

And as ac instruments reads rms value, the reading of ammeter will be,

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_0}{\sqrt{2}X_C} \quad \left[\text{as } E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

i.e. $I_{rms} = \frac{200\sqrt{2}}{\sqrt{2}\times 10^4} = 20\text{mA}$ **Ans**

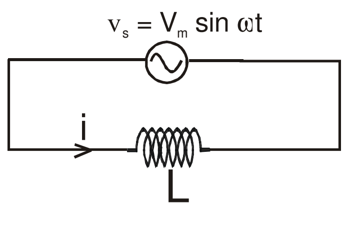
Ques. A 10 μF capacitor is connected with an ac source $E = 200\sqrt{2}\sin(100\,t)$ V through an ac ammeter (it reads rms value) . What will be the reading of the ammeter?

Ans: 200 mA

Ques. Find the reactance of a capacitor (C = 200 μF) when it is connected to (a) 10 Hz AC source, (b) a 50 Hz AC source and (c) a 500 Hz AC source.

Ans. (a) 80 Ω for 10 Hz AC source, (b) 16 Ω for 50 Hz and (c) 1.6 Ω for 500 Hz.

9. PURELY INDUCTIVE CIRCUIT:



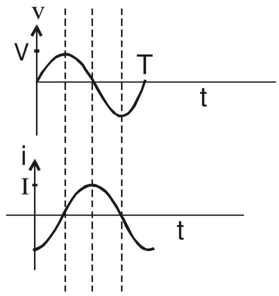
Writing KVL along the circuit,

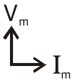
$$v_s - L\frac{di}{dt} = 0 \quad \Rightarrow \quad L\frac{di}{dt} = V_m \sin\omega t$$

$$\int L\,di = \int V_m \sin\omega t\,dt \qquad i = -\frac{V_m}{\omega L}\cos\omega t + C$$

$< i > = 0 \quad \Rightarrow \quad C = 0$

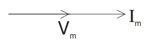
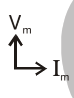
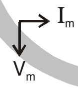
$\therefore \; i = -\dfrac{V_m}{\omega L}\cos\omega t \quad \Rightarrow \quad I_m = \dfrac{V_m}{X_L}$



From the graph of current versus time and voltage versus time , it is clear that voltage attains its peak value at a time $\frac{T}{4}$ before the time at which current attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference = $\omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. Diagrammatically (phasor diagram) it is repre-sented as  . i_L lags behind v_L by $\pi/2$.

Since $\phi = 90^\circ$, $<P> = V_{rms}I_{rms}\cos\phi = 0$

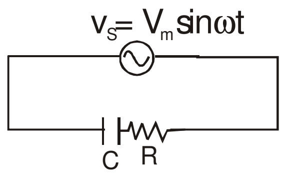
Summary :

AC source connected with	ϕ		Z	Phasor Diagram
Pure Resistor	0	v_R is in same phase with i_R	R	
Pure Inductor	$\pi/2$	v_L leads i_L	X_L	
Pure Capacitor	$\pi/2$	v_C leads i_C	X_C	

Ques. An inductor (L = 200 mH) is connected to an AC source of peak current. What is the intantaneous voltage of the source when the current is at its peak value?

Ans. zero

10. RC SERIES CIRCUIT WITH AN AC SOURCE :



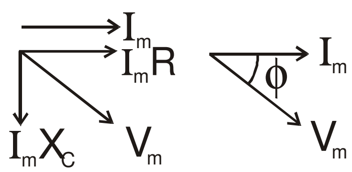
Let $i = I_m \sin(\omega t + \phi) \quad \Rightarrow \quad v_R = iR = I_m R \sin(\omega t + \phi)$

$v_C = I_m X_C \sin(\omega t + \phi - \frac{\pi}{2}) \quad \Rightarrow \quad v_S = v_R + v_C$

or $\quad V_m \sin(\omega t + \phi) = I_m R \sin(\omega t + \phi) + I_m X_C \sin(\omega t + \phi - \frac{\pi}{2})$

$$V_m = \sqrt{(I_m R)^2 + (I_m X_C)^2 + 2(I_m R)(I_m X_C)\cos\frac{\pi}{2}}$$

OR $\quad I_m = \dfrac{V_m}{\sqrt{R^2 + X_C^2}} \quad \Rightarrow \quad Z = \sqrt{R^2 + X_C^2}$



Using phasor diagram also we can find the above result.

$\tan\phi = \dfrac{I_m X_C}{I_m R} = \dfrac{X_C}{R}$.

Ques. An AC source producing emf $\xi = \xi_0[\cos(100\,\pi\,s^{-1})t + \cos(500\,\pi\,s^{-1})t]$ is connected in series with a capacitor and a resistor. The steady-state current in the circuit is found to

Teko Classes, Maths : Suhag R. Kariya (S. R. K. Sir), Bhopal Phone : 0 903 903 7779, 0 98930 58881.

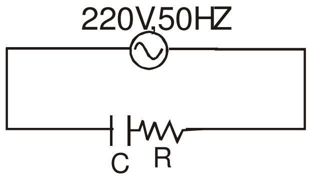
be $i = i_1 \cos[(100\pi\, s^{-1})t + \varphi_1] + i_2 \cos[(500\pi\, s^{-1})t + \varphi_1]$

(A) $i_1 > i_2$ (B) $i_1 = i_2$ (C) $i_1 < i_2$

(D) the information is insufficient to find the relation between i_1 and i_2

Ans. (C)

Ex. 11 In an RC series circuit, the rms voltage of source is 200V and its frequency is 50 Hz. If R = 100 Ω and C = $\frac{100}{\pi}$ μF, find



(i)	Impedance of the circuit	(ii)	Power factor angle
(iii)	Power factor	(iv)	Current
(v)	Maximum current	(vi)	voltage across R
(vii)	voltage across C	(viii)	max voltage across R
(ix)	max voltage across C	(x)	< P >
(xi)	$< P_R >$	(xii)	$< P_C >$

Sol. $X_C = \dfrac{10^6}{\frac{100}{\pi}(2\pi 50)} = 100\ \Omega$

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\ \Omega$ (ii) $\tan\phi = \dfrac{X_C}{R} = 1$ $\therefore\ \phi = 45^\circ$

(iii) Power factor $= \cos\phi = \dfrac{1}{\sqrt{2}}$ (iv) Current $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{200}{100\sqrt{2}} = \sqrt{2}$ A

(v) Maximum current $= I_{rms}\sqrt{2} = 2$A

(vi) voltage across R $= V_{R,rms} = I_{rms}R = \sqrt{2} \times 100$ Volt

(vii) voltage across C $= V_{C,rms} = I_{rms}X_C = \sqrt{2} \times 100$ Volt

(viii) max voltage across R $= \sqrt{2}\ V_{R,rms} = 200$ Volt

(ix) max voltage across C $= \sqrt{2}\ V_{C,rms} = 200$ Volt

(x) $<P> = V_{rms}I_{rms}\cos\phi = 200 \times \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 200$ Watt

(xi) $<P_R> = I_{rms}^2 R = 200$ W (x) $<P_C> = 0$

Ex. 12 In the above question if $v_s(t) = 220\sqrt{2}\ \sin(2\pi\, 50\, t)$, find (a) i (t), (b) v_R and (c) v_C (t)

Sol. (a) $i(t) = I_m \sin(\omega t + \phi)$ $= \sqrt{2}\ \sin(2\pi\ 50\ t + 45^\circ)$

(b) $v_R = i_R \cdot R = i(t)\, R$ $= \sqrt{2} \times 100 \sin(100\pi t + 45^\circ)$

(c) $v_C(t) = i_C X_C$ (with a phase lag of 90º) $= \sqrt{2} \times 100 \sin(100\pi t + 45 - 90)$

Ex. 13 An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to ω/3 (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω.

Sol. According to given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \quad \dots (1)$$

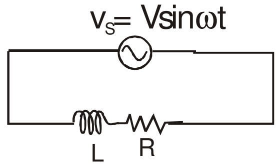
and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \quad \dots (2)$$

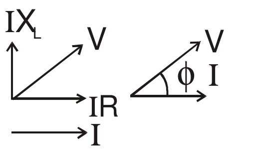
Substituting the value of I from Equation (1) in (2),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}. \text{ i.e., } \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that, $\dfrac{X}{R} = \dfrac{(1/C\omega)}{R} = \dfrac{\left(\dfrac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\dfrac{3}{5}}$ **Ans.**

11. LR SERIES CIRCUIT WITH AN AC SOURCE :





From the phasor diagram

$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = I\,Z$

$\tan\phi = \dfrac{IX_L}{IR} = \dfrac{X_L}{R}$

Ex. 14 A $\dfrac{9}{100\pi}$ H inductor and a 12 ohm resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

Sol. Here $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \dfrac{9}{100\pi} = 9\ \Omega$

So, $Z = \sqrt{R^2 + X_L^{\,2}} = \sqrt{12^2 + 9^2} = 15\ \Omega$

So (a) $I = \dfrac{V}{Z} = \dfrac{225}{15} = 15$ A **Ans**

and (b) $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{9}{12}\right) = \tan^{-1} 3/4 = 37^\circ$

i.e., the current will lag the applied voltage by 37º in phase. **Ans**

Ex. 15 When an inductor coil is connected to an ideal battery of emf 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an AC source of 10 V and 50 Hz then the current is 2A. Find out inductance of the coil .

Sol. When the coil is connected to dc source, the final current is decided by the resistance of the coil .

$\therefore \quad r = \dfrac{10}{2.5} = 4\ \Omega$

When the coil is connected to ac source, the final current is decided by the impedance of the coil .

$\therefore \quad Z = \dfrac{10}{2} = 5\,\Omega$

But $\quad Z = \sqrt{(r)^2 + (X_L)^2} \qquad X_L^{\,2} = 5^2 - 4^2 = 9$

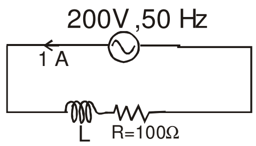
$X_L = 3\ \Omega$

$\therefore \quad \omega L = 2\pi f L = 3 \qquad \therefore \quad 2\pi\, 50\, L = 3$

$\therefore \quad L = 3/100\pi$ Henry

Ex. 16 A bulb is rated at 100 V,100 W , it can be treated as a resistor .Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

Sol: From the rating of the bulb , the resistance of the bulb is $R = \dfrac{V_{rms}^{\ 2}}{P} = 100\ \Omega$



For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $I_{rms} = \frac{V_{rms}}{Z}$ $\quad\therefore\quad$ $1 = \frac{200}{\sqrt{100^2 + (2\pi 50 L)^2}}$ $\quad$ $L = \frac{\sqrt{3}}{\pi}$ H

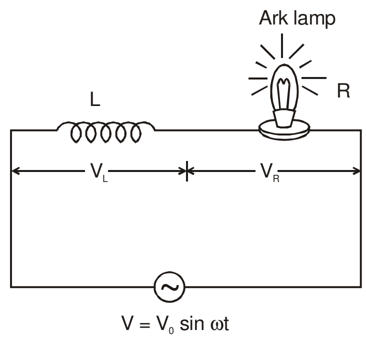
Ex. 17 A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both cases.

Sol. As for lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,

$V^2 = V_R^{\,2} + V_L^{\,2}$, $\qquad V_L = \sqrt{160^2 - 50^2} = 152$ V

and as, $\qquad V_L = IX_L = I\omega L = 2\pi f L I$

So, $L = \frac{V_L}{2\pi f I} = \frac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H **Ans.**



Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V = I(R + r)$, i.e., $160 = 10(5 + r)$

i.e., $\quad r = 11\ \Omega$ **Ans.**

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

$P = I^2 R = 10^2 \times 5 = 500$ W

However, in case of dc as resistance r is to be used instead of choke, the power loss in the resistance r will be.

$PL = 10^2 \times 11 = 1100$ W

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

Ques. An alternating voltage of 220 volt r.m.s. at a frequency of 40 cycles/sec is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 ohms in series. Calculate (i) the current, (ii) potential difference across the resistance, (iii) potential difference across the inductance, (iv) the time lag, (v) power factor.

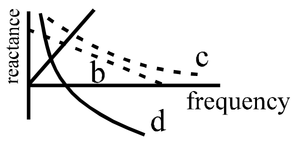
Ans. (i) 33.83 amp. (ii) 202.98 volts (iii) 96.83 volts (iv) 0.01579 sec (v) 0.92

12. LC SERIES CIRCUIT WITH AN AC SOURCE :

From the phasor diagram

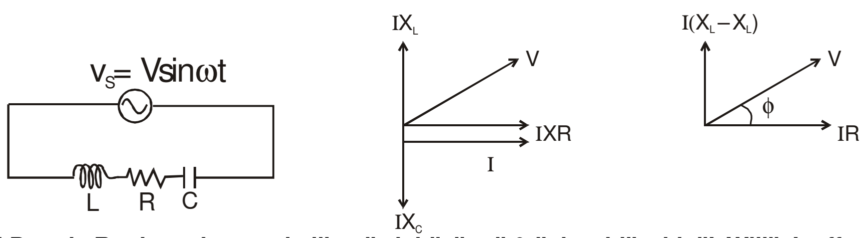
$V = I\left|(X_L - X_C)\right| = I\,Z \qquad \phi = 90^\circ$

Ques. Which of the following plots may represnet the reactance of a series LC combination ?



Ans. D

13. RLC SERIES CIRCUIT WITH AN AC SOURCE :



From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ \qquad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

Ques. A series AC circuit has resistance of 4 Ω and a reactance of 3 Ω. the impedance of the circuit is

(A) 5 Ω (B) 7 Ω (C) 12/7 Ω (D) 7/12 Ω

Ans. (A)

13.1 Resonance :

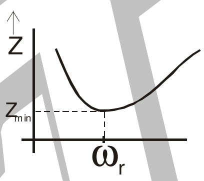
Amplitude of current (and therefore I_{rms} also) in an RLC series circuit is maximum for a given value of V_m and R , if the impedance of the circuit is minimum, which will be when X_L-X_C =0. This condition is called **resonance**.

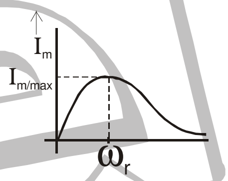
So at resonance:

$X_L - X_C = 0$.

or $\omega L = \dfrac{1}{\omega C}$

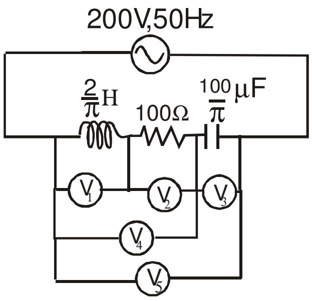
or $\omega = \dfrac{1}{\sqrt{LC}}$. Let us denote this ω as ω_r.





Ex. 18 In the circuit shown in the figure , find

(a) the reactance of the circuit .
(b) impedance of the circuit
(c) the current
(d) readings of the ideal AC voltmeters (these are hot wire instruments and read rms values).



Sol: (a) $X_L = 2\pi f L = 2\pi \times 50 \times \dfrac{2}{\pi} = 200\ \Omega$

$$X_C = \frac{1}{2\pi 50 \dfrac{100}{\pi} \times 10^{-6}} = 100\ \Omega$$

∴ The reactance of the circuit $X = X_L - X_C = 200 - 100 = 100\ \Omega$

Since $X_L > X_C$,the circuit is called inductive.

(b) impedance of the circuit $Z = \sqrt{R^2 + X^2} = \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega$

(c) the current $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{200}{100\sqrt{2}} = \sqrt{2}$ A

(d) readings of the ideal voltmeter

V_1: $I_{rms}X_L = 200\sqrt{2}$ Volt

V_2: $I_{rms}R = 100\sqrt{2}$ Volt

V_3: $I_{rms}X_c = 100\sqrt{2}$ Volt

V_4: $I_{rms}\sqrt{R^2 + X_L^2} = 100\sqrt{10}$ Volt

V_5: $I_{rms}Z = 200$ Volt ,which also happens to be the voltage of source.

13.1 Q VALUE (QUALITY FACTOR) OF LCR SERIES CIRCUIT (NOT IN IIT SYLLABUS) :

Q value is defined as $\frac{X_L}{R}$ where X_L is the inductive reactance of the circuit, at resonance.

More Q value implies more sharpness of I Vs ω curve